TRIGONOMETRIČNE ENAČBE

ENAČBE OBLIKE $a\sin x + b\cos x = 0$

$\sin x + \cos x = 0 \quad / : \cos x$

$\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x} = 0$

$\tan x + 1 = 0$
$\tan x = -1$
$x = \arctan(-1) + k\pi$
$x = -\frac{\pi}{4} + k\pi;\ k \in \mathbb{Z}$

znamzasi

Deliš s cosx, da dobiš tanx.

$\tan x = \frac{\sin x}{\cos x}$

$\tan x = a$
$x = \arctan a + k\pi;\ k \in \mathbb{Z}$

$\cos x - \sqrt{3}\sin x = 0 \quad / : \cos x$

$1 - \sqrt{3}\frac{\sin x}{\cos x} = 0$

$1 - \sqrt{3}\tan x = 0$
$-\sqrt{3}\tan x = -1 \quad / : (-\sqrt{3})$

$\tan x = \frac{1}{\sqrt{3}}$
$x = \arctan\frac{1}{\sqrt{3}} + k\pi$
$x = \frac{\pi}{6} + k\pi;\ k \in \mathbb{Z}$

$\sin x - \sqrt{3}\cos x = 0 \quad / : \cos x$

$\frac{\sin x}{\cos x} - \sqrt{3} = 0$
$\tan x - \sqrt{3} = 0$

$\tan x = \sqrt{3}$
$x = \arctan\sqrt{3} + k\pi$
$x = \frac{\pi}{3} + k\pi;\ k \in \mathbb{Z}$

$5\sin x + 2\cos x = 0 \quad / : \cos x$

$5\tan x + 2 = 0$
$5\tan x = -2 \quad / : 5$

$\tan x = -\frac{2}{5}$
$x = \arctan\left(-\frac{2}{5}\right) + k\pi;\ k \in \mathbb{Z}$

(pustiš takšen rezultat ali napišeš približek
PAZI, da je kalkulator v radianih!
$x = -0{\cdot}38 + k\pi;\ k \in \mathbb{Z}$

ENAČBE OBLIKE $a\sin x + b\cos x = c$

Rešiš jih lahko s polovičnimi koti.

$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$
$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$
$c = c\cdot 1 = c\cdot(\sin^2\frac{x}{2} + \cos^2\frac{x}{2})$

$\sin x + \cos x = 1$ — $\sin^2\frac{x}{2} + \cos^2\frac{x}{2}$

$2\sin\frac{x}{2}\cos\frac{x}{2}$ $\quad$ $\cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$2\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \sin^2\frac{x}{2} + \cos^2\frac{x}{2}$ daš vse na eno stran

$2\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2} - \sin^2\frac{x}{2} - \sin^2\frac{x}{2} - \cos^2\frac{x}{2} = 0$

$2\sin\frac{x}{2}\cos\frac{x}{2} - 2\sin^2\frac{x}{2} = 0 \;/:2$

$\sin\frac{x}{2}\cos\frac{x}{2} - \sin^2\frac{x}{2} = 0$ izpostaviš $\sin\frac{x}{2}$

$\sin\frac{x}{2}(\cos\frac{x}{2} - \sin\frac{x}{2}) = 0$

$\sin\frac{x}{2} = 0$

$\frac{x}{2} = 0 + k\pi \;/\cdot 2$

$\underline{x_1 = 2k\pi}$

$\cos\frac{x}{2} - \sin\frac{x}{2} = 0 \;/:\cos\frac{x}{2}$

$1 - \tan\frac{x}{2} = 0$

$\tan\frac{x}{2} = 1$

$\frac{x}{2} = \frac{\pi}{4} + k\pi \;/\cdot 2$

$\underline{x_2 = \frac{\pi}{2} + 2k\pi;\; k\in\mathbb{Z}}$

$3\cos x + 4\sin x - 5 = 0$

$3(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}) + 4\cdot 2\sin\frac{x}{2}\cos\frac{x}{2} - 5(\sin^2\frac{x}{2} + \cos^2\frac{x}{2}) = 0$

$3\cos^2\frac{x}{2} - 3\sin^2\frac{x}{2} + 8\sin\frac{x}{2}\cos\frac{x}{2} - 5\sin^2\frac{x}{2} - 5\cos^2\frac{x}{2} = 0$

$-8\sin^2\frac{x}{2} + 8\sin\frac{x}{2}\cos\frac{x}{2} - 2\cos^2\frac{x}{2} = 0 \;/:(-2)$

$4\sin^2\frac{x}{2} - 4\sin\frac{x}{2}\cos\frac{x}{2} + \cos^2\frac{x}{2} = 0 \;/:\cos^2\frac{x}{2}$ (dobiš kvadratno enačbo s $\tan\frac{x}{2}$)

$$4\frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}} - 4\frac{\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}} + \frac{\cos^2\frac{x}{2}}{\cos^2\frac{x}{2}} = 0$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$4\tan^2\frac{x}{2} - 4\tan\frac{x}{2} + 1 = 0$$

$$\tan\frac{x}{2} = t$$

$$4t^2 - 4t + 1 = 0$$

a b c

$$D = (-4)^2 - 4\cdot 4\cdot 1 = 0$$

$$t_{1,2} = \frac{4 \pm 0}{2\cdot 4} = \frac{1}{2}$$

$$D = b^2 - 4ac$$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

$$\tan\frac{x}{2} = \frac{1}{2}$$

$$\frac{x}{2} = \arctan\frac{1}{2} + k\pi \quad / \cdot 2$$

$$x = 2\arctan\frac{1}{2} + 2k\pi;\ k \in \mathbb{Z}$$

znamzasi